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Role of the temperature in the chaotic behavior of a p–n junction

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Abstract

We have measured the temperature dependence of first-return maps for the maximum forward current through a driven diode oscillator circuit. We have constructed the bifurcation diagram with temperature as the control parameter and generalized the model of Rollins and Hunt to include the temperature.

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1. Introduction

The chaotic behavior of a driven diode in series with an inductance and a resistance has been extensively studied [1–11]. The first experimental study of the period doubling route to chaos in this system was done by Linsay [1]. He used a varactor diode, in which nonlinearity arises because the junction capacitance varies as a function of the voltage drop. The subharmonic spectrum for successive period doubling was obtained with the amplitude of the driving voltage as the bifurcation parameter. The chaotic behavior was explicitly related to the nonlinear junction capacitance. Based upon detailed measurements on a driven Si varactor diode, Testa et al. [2] established a quantitative connection between their experimental results and the period doubling cascade of the logistic map. Hunt proposed [3], and Rollins and Hunt [4] solved a model of a diode driven by a sinusoidal voltage $V(t)$ in series with an inductor L and a resistor R. They found a one-dimensional map exhibiting the period doubling route to chaos. In this model when the diode is conducting, it is replaced by an emf $= V_f$, and when it is not conducting, the diode acts as a fixed capacitance C. The property responsible for the chaotic behavior is taken to be the reverse recovery time of the junction. Changes in both the capacitance and the voltage drop across the internal resistance of the diode are neglected. The response of the system is studied as a function of the amplitude V_0 of the driving voltage. A generalization of the above model was later reported by the same authors [5]. This modification takes into account the memory of the system and can be used to explain features of experimental return maps which cannot be accounted for by one-dimensional maps [12].

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The purpose of this Letter is to show, for the first time, that the temperature T can be used as a bifurcation parameter and that the effect of temperature can be understood in terms of a simple generalization of the model of Rollins and Hunt [4].

2. Experimental setup and results

In order to study the role of the temperature in the chaotic dynamics of the anharmonic oscillator we used a silicon general purpose diode 1N4005 in series with a 3.8 mH inductor and a 40 Ω resistance, sinusoidally driven by a hp3312A function generator at a frequency of 160 kHz and a peak to peak voltage of 22 V. An oscilloscope Philips model 3335 (60 MHz), with memory, monitors the voltage drop through the resistance, and sends a time series of 8192 points to a Macintosh personal computer. For this setting the system behaves chaotically at room temperature. When the temperature of the diode is lowered, the system's behavior changes, going through different periodic windows. A diagram of the experimental setup is shown in Fig. 1.

To measure the temperature we used a platinum resistance thermometer connected to a Keithley multimeter model 199. The thermometer is placed as close as possible to the diode. To lower the temperature we use a dewar storage container with liquid nitrogen, as is schematically shown in Fig. 2. The level of the liquid nitrogen is maintained at about one fifth of the total depth of the container. In this way a temperature gradient is established along the container. The diode, together with the resistance thermometer, is placed on the axis of the cylindrical container, at a certain distance from the level of the liquid nitrogen. To control the temperature we adjust the vertical position of the diode. To heat the diode we used a home made cylindrical oven controlled with a Variac. The temperature is controlled varying the position of the diode and the current through the heating coils. The fluctuations in temperature are maintained within 1 K.

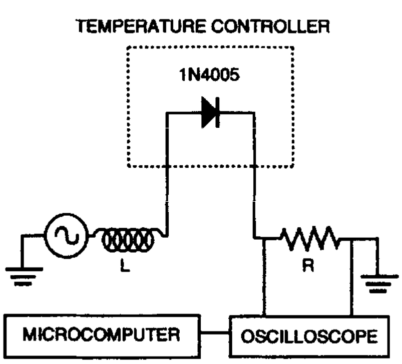

Fig. 1. Experimental apparatus used to obtain the return maps for various temperatures, and the bifurcation diagram of the maximum forward current I_m as a function of temperature T.

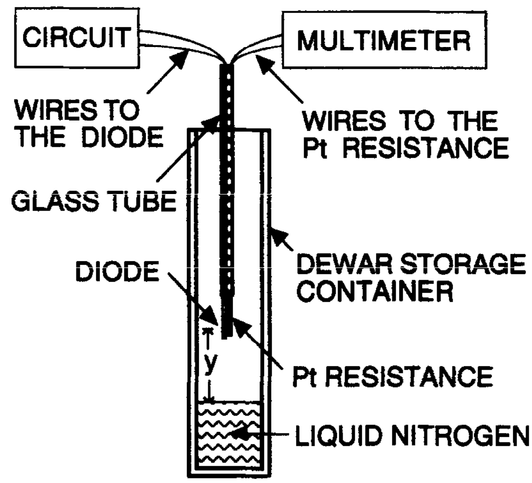

Fig. 2. Sketch of the temperature control system for measurements below room temperature.

In Fig. 3 we show the return maps for the current, $|I_{n+1}|$ versus $|I_n|$, for two different temperatures. The shape of the map shown in Fig. 3a is almost identical to the one obtained by Hunt and Rollins [5], just below the period-three window, for a similar rectifier diode. Our experimental return map is also nearly identical to the one obtained by them from a

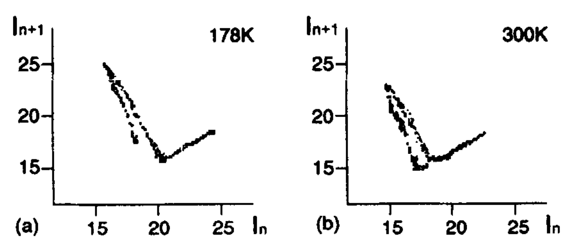

Fig. 3. (a) Return map for the current, before the period three window at 178 K. (b) Return map after the period three cascade at 300 K. The current is given in mA.

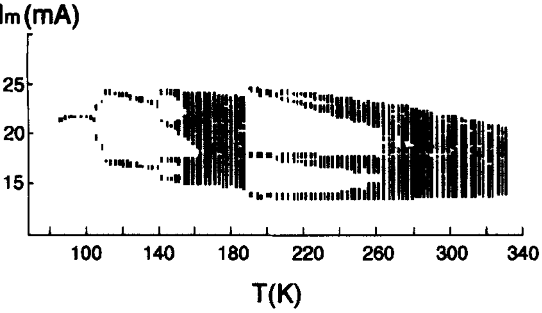

Fig. 4. Experimental bifurcation diagram of the maximum forward current I_m as a function of temperature. This was varied between 80 and 336 K.

simulation, using their modified model [5]. The maps shown in Fig. 3 are quite similar to others reported in the literature [5,8]. They show features of multidimensional maps, are highly asymmetric, and have cusp-like maxima. This makes them qualitatively different from the logistic map. These features have been discussed by several authors [5,8,10,13], but we will not consider them in this paper.

In Fig. 4 we show the measured bifurcation diagram for the absolute value of the maximum forward current $|I_m|$ as a function of temperature of the diode. This map of the subharmonic cascade was obtained from the projection of 122 different return maps. Each one of them is made from 300 points. The map shown is for one set of temperatures, going up. We have made several bifurcation diagrams raising the temperature, and several ones lowering the temperature. We have found that the maps are nearly identical, provided that enough time has expired, at a certain temperature, before registering the time series. We wait five minutes at a given temperature, before registering the time series for the current through the diode.

The diode used, at room temperature, has a reverse recovery time comparable to the period of the driving frequency, and is not a varactor diode. We chose to measure the reverse recovery time as a function of temperature to determine if the response of the junction, to the change of polarity of the driving voltage, could be properly described by the following simple saturation function of the maximum forward current [14],

$$\tau_r(T) = \tau_m(T)[1 - e^{-|I_m(T)|/I_c(T)}], \qquad (1)$$

where $\tau_r(T)$ is the reverse recovery time at temperature T, I_m is the maximum forward current through the diode, just before the driving voltage changes polarity, τ_m is the saturation time and I_c is a parameter that depends on the specific diode used.

Eq. (1) corresponds to the expression assumed by Rollins and Hunt, where we have added the possible temperature T dependence of the parameters involved. To obtain these parameters experimentally the junction was driven by a square wave, with a period much larger than the reverse recovery time [15]. The time τ_r was measured directly on an oscilloscope screen, as a function of the maximum forward current I_m, for different temperatures.

In Fig. 5 we show a plot of the data for τ_r versus I_m for different temperatures. At a given temperature, I_m was first increased and then decreased. In the graph shown each point represents the average of

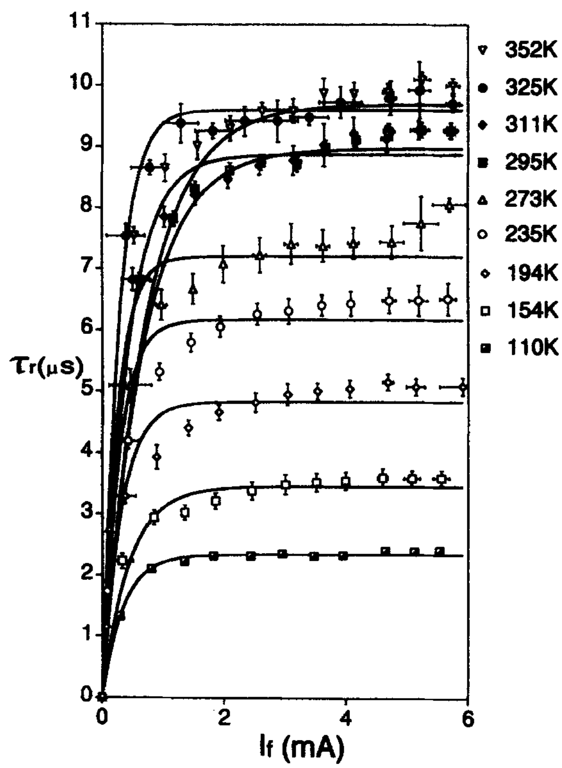

Fig. 5. Reverse recovery time as a function of maximum forward current through the diode for different temperatures. The continuous line represents the fitting of the two-parameter function given by Eq. (1) to the experimental data, using simulated annealing.

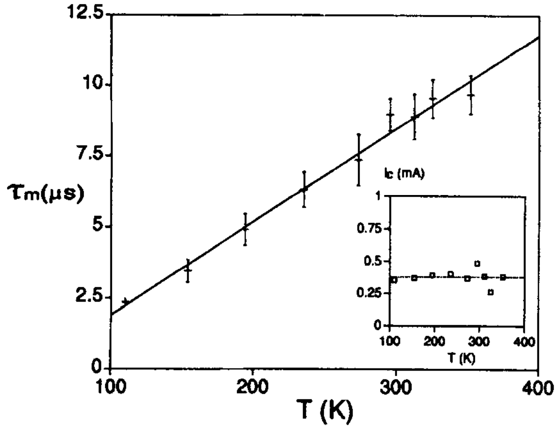

Fig. 6. Linear least squares fit for $\tau_m(T)$. In the inset we show the data for $I_c(T)$. The dotted line corresponds to the average value of I_c.

six measurements. The parameters τ_m and I_c, for each temperature, are obtained fitting the data to the functional relationship for τ_r given by Eq. (1). The fit shown was done using simulated annealing. Some curves were double checked using the downhill simplex method [16]. A reasonable fit of the data in terms of the chosen saturating function of the maximum forward current through the diode was obtained. From the data shown in Fig. 5 we see that the model progressively fails at higher temperatures. More measurements on several nominally identical diodes are necessary to elucidate the nature of the observed discrepancy.

In Fig. 6 we show a plot of the resultant fit for the parameter τ_m and I_c, as a function of temperature. According to our results I_c is weakly dependent on temperature and can be taken as a constant, and τ_m has a linear dependence on T. In the assumed model the capacitance is considered a constant. This simplifying assumption is motivated by the following: (a) We are not using a varactor diode, (b) we are not changing the driving voltage, and (c) we do not expect the junction capacitance to change significantly with temperature, in the range of temperature that we used in our experiment. The temperature dependence for the junction capacitance of the diode, is of the following form, $C \propto (kT)^{5/4} \times \exp(-E_g/2kT)$, where k is the Boltzmann constant and E_g is the energy gap of the p–n junction [17]. This function is approximately constant in the temperature range that we measured.

From the fit of the data, shown in Fig. 6, we obtain the following empirical equation for the temperature dependence of the reverse recovery time,

$$\tau_r(T, I_m) = (\alpha T - \tau_0)(1 - e^{-I_m(T)I/I_c}), \qquad (2)$$

where $\alpha = (\partial \tau_m / \partial T) = (3.3 \pm 0.3) \times 10^{-2}$ μs/K, $\tau_0 = 1.4 \pm 0.7$ μs, $I_c = 0.36 \pm 0.09$ mA.

Eq. (2) indicates that the reverse recovery time can be changed in the following ways: (a) Changing the amplitude of the driving voltage, which changes I_m, and (b) changing the temperature of the diode, which changes τ_m and I_m.

We found that the maximum forward current can change via the temperature dependence of the emf V_f. The measured temperature dependence of V_d is shown in Fig. 7. From the linear least squares fit shown we obtain the following empirical equation for the voltage drop across the diode,

$$V_d(T, I) = -[\beta T + V_1 + V_2(I)], \qquad (3)$$

where $\beta = -(\partial V_f / \partial T) = 1.55 \pm 0.06$ mV/K, V_1 is a voltage independent of T and I, $V_2 = IR_d$, R_d is the internal resistance of the diode when it is conducting, $V_1 + V_2(I) = -(1.17 \pm 0.01) \times 10^3$ mV.

We neglect the voltage drop $V_2(I)$ across the internal resistance R_d of the diode. The current dependent voltage drop was at least two orders of

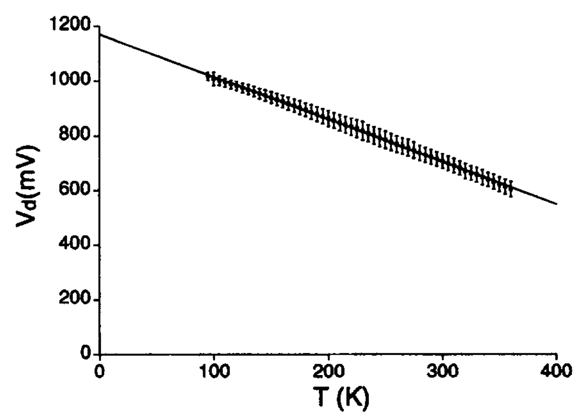

Fig. 7. Fit for the voltage drop across the diode versus temperature, when it is conducting.

magnitude smaller than the current independent part. Therefore we can write the following equation,

$$V_d(t,T) = -V_f(T), \tag{4}$$

where

$$V_f(T) = \beta T + V_1. \tag{5}$$

Consequently, the resultant expression for the current, when the diode is conducting, is given by the following equation [4],

$$I(t,T;I_0) = (V_0/Z_a)\cos(\omega t - \theta_a) + I_0 e^{-Rt/L} + V_f(T)/R, \tag{6}$$

where $I(t,T;I_0)$ is the current trough the diode, when it is conducting, $Z_a = R^2 + (\omega L)^2$, ω is the angular frequency of the driving voltage, $\theta_a = \arctan(\omega L/R)$, I_0 is a constant that depends on the initial conditions for each conducting cycle. The temperature dependence of the forward current enters only through the forward voltage $V_f(T)$.

In this model, the charge storage capacitance [18] does not appear in the equation for the forward current. A deeper study of the physical properties that give rise to the chaotic behavior of p–n junctions would have to include this property of the diode. For temperatures higher than the ones we measured, the junction capacitance becomes highly dependent on temperature. Under these conditions the variation of the capacitance with temperature would have to be taken into account.

3. Analysis and discussion

From Eqs. (5) and (6) we can see that the current varies linearly with the driving voltage and with the temperature, consequently it can be expressed in the following way,

$$I = AV_0 + BT + D, \tag{8}$$

where, A, B, D are independent of V_0 and T. This implies that the current can be varied via these two independent parameters, and explains why the resultant route to chaos is qualitatively similar in both cases. The nonlinearity arises from Eq. (2). These experimental results provide a two-parameter space that can be explored. This can be particularly important in electronic circuit design. Furthermore, the temperature can be a powerful probe for establishing the connection between the chaotic behavior of the diode and its detailed internal dynamics. Another implication of our experimental results is that the model of Rollins and Hunt yields a good representation for the chosen system.

We are presently carrying out simulations to include the temperature dependence of the reverse recovery time. This will be reported in a future article.

4. Conclusions

In conclusion, we have measured, for the first time, the period doubling cascade, with the temperature as an independent control parameter, for a driven diode, in series with an inductor and a resistor. We have generalized the model of Rollins and Hunt to include the temperature.

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