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Onset of Turbulence in Long Josephson Junctions

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Abstract

The r.f.-biased long Josephson junction behave very much like turbulent systems as characterized by the absence of spatial correlation for a wide range of frequencies. We show that this system approaches the turbulent state through a well defined transition beyond a quasiperiodic regime, which is intimately related to spatiotemporal symmetry-breaking and to the existence and breakdown of spontaneous pattern formation. We also show that the local generation of different linear combinations of frequencies in the quasiperiodic regime leads to the breakdown of coherence of the spatiotemporal profile.

1. Introduction

Turbulent behavior is an interesting and fundamental dynamical state not well understood. Once it had been established that *chaos is not turbulence*, the turbulent phenomenon has received a great deal of attention in recent years [1–7].

Systems with many degrees of freedom can develop coherent structures (spatiotemporal patterns). In this way the system reduces the effective number of degrees of freedom and is able to exhibit periodic and low-dimensional chaotic (*in time*) behavior. The activation of new degrees of freedom can give rise to turbulent-like dynamics (incoherent in space and *disordered in space as well as in time*).

The study of the route to turbulent-like behavior can establish what determines the ability of the system to become turbulent, providing in this fashion a better understanding of the turbulent state.

In this paper we explore the onset of turbulence in a solid-state device, the long Josephson junction (LJJ) as the amplitude of a radio-frequency (r.f.) drive is increases.

We show that the onset of the turbulent-like regime is preceded by a spatial bifurcation: after a transient homogeneous in space and with period 1 in time, the system oscillates with period 2 between two different structures in space. As the r.f. drive is increased this periodic regime is replaced by a quasiperiodic one which happens because the spatiotemporal excitation oscillates at a frequency different from the driving frequency. We show for this quasiperiodic regime that different local generation of linear combinations of the two basic frequencies (the driving frequency and the one of the spatiotemporal excitation) affect the coherence of the spatiotemporal profile. Finally, we describe the breakdown of coherence of the spatiotemporal profile as the system is driven harder.

Our paper is organized as follows: in Section 2 we present a description of our model and simulation. In Section 3 we present the spontaneous development of a spatiotemporal profile (locked to the drive)in the system. In Section 4 we present the unlocking of the spatiotemporal profile to drive and its breakdown of coherence as the amplitude of the r.f. drive is increased. In Section 5 we summarize and present our conclusions.

2. The long Josephson junction

The forced LJJ considered by us has been discussed extensively [8]. We model this system with the usual sine-Gordon-like equation,

$$\phi_{xx} - \phi_{tt} - \sin\phi = \alpha\phi_t - \rho\sin(\Omega_d t) \quad (1)$$

where $\phi = \phi(x, t)$ is the phase difference of the superconducting order parameter between each side of the barrier and its derivative in time is the voltage across the junction. The term $\alpha\phi_t$ represents quasiparticle loss. The distance is normalized to the Josephson penetration depth λ_J, time is normalized to the inverse of the Josephson plasma frequency, the r.f. amplitude ρ is normalized to the critical current current and Ω_d is the normalized applied frequency.

The external applied field is taken into account through,

$$\phi_x(0, t) = \phi_x(L, t) = \eta \quad (2)$$

where L is the junction length and η is a measure of the external magnetic field. In this paper parameter values are $L = 10\lambda_J$, $\alpha = 0.252$ and $\Omega_d = 0.65$. We employ open boundary condition, $\eta = 0.0$, spatially uniform drive and flat initial conditions thus the pattern formation phenomena is fully spontaneous in contrast with Rayleigh–Bénard experiments in which symmetry breaking and consequent pattern formation is induced via boundary condition. In addition, our results were obtained in the absence of thermal noise.

The perturbed sine-Gordon equation is the simplest wave-equation for a periodic medium occurs frequently in solid-state physics. The system described by eqs. (1, 2) is in fact analogous to a chain of coupled pendula forced by an external torque. This analogy indicates that the turbulent behavior can be present not only in fluids but also in solid-state and mechanical systems described by relatively simple models.

3. Spontaneous pattern formation and symmetry breaking

In most physical situations the development of turbulence is preceded by the formation of spatiotemporal structures. In this section we show that in the case of the LJJ the pattern formation phenomenon can be autonomously excited.

For a LJJ with $\rho = 2$ we find a transition from a transient with period 1 to a period 2 regime, as can be appreciated in the strobed time series of Fig. 1(a) in which the value of the voltage ϕ_t is plotted at each period of the forcing. This transition at first sight might appear to be a simple bifurcation in time as in low-dimensional systems. However, as shown in Fig. 1(b) (where we plot the phase difference $\Delta\phi$ between two points of the junction for each period of the r.f. drive as a function of time) the voltage is no longer homogeneous in space. The fact that $\Delta\phi$ repeats ony every second period shows that the spatial extent of the junction has now become significant. Figure 1(c)

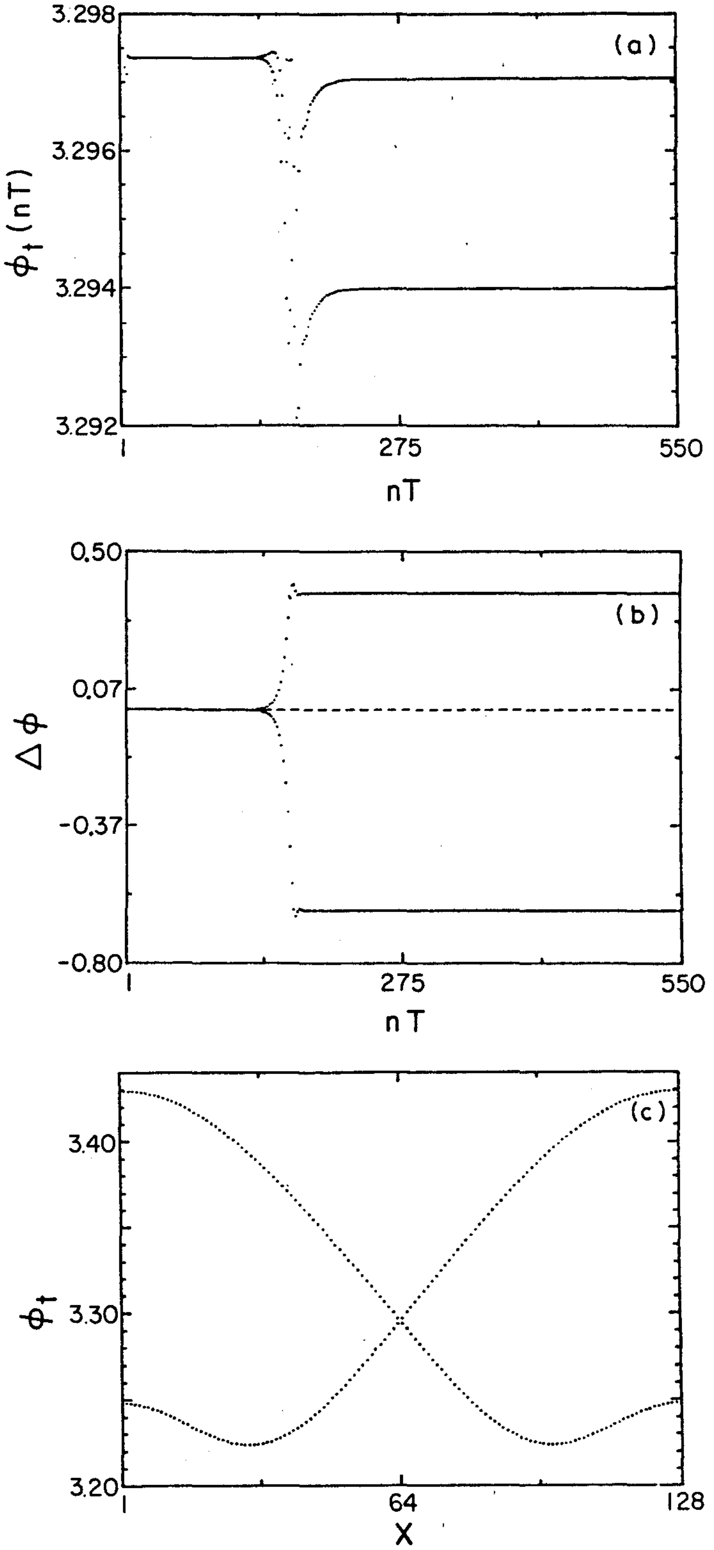

Fig. 1. Spontaneous Symmetry Breaking. (a) Strobed time series $\phi_t(nT)$ vs. nT (T is the period of the harmonic drive) at the center of the junction showing a transition between period 1 and period 2 solutions ($\rho = 2$). For $n > 180$, ϕ_t oscillates between the upper and lower curves. (b) The difference of the phase between two points of the junction ($\Delta\phi(nT) = \phi(L/2, nT) - \phi(L', nT)$) vs. nT, reveals that the transition is associated with the development of a spatiotemporal profile, (c) Strobed profile $\phi_t(x, nT)$ vs. x showing nonsymmetric breather oscillation.

reveals the spatiotemporal profile sustained by the system after the homogeneous transient: a "virtual" breather (a solitonlike state with an internal degree of freedom) centered at the edge of the junction is formed with only half of it present inside the junction. This "virtual" breather switches at each period between its two states, thus yielding a steady state in which the

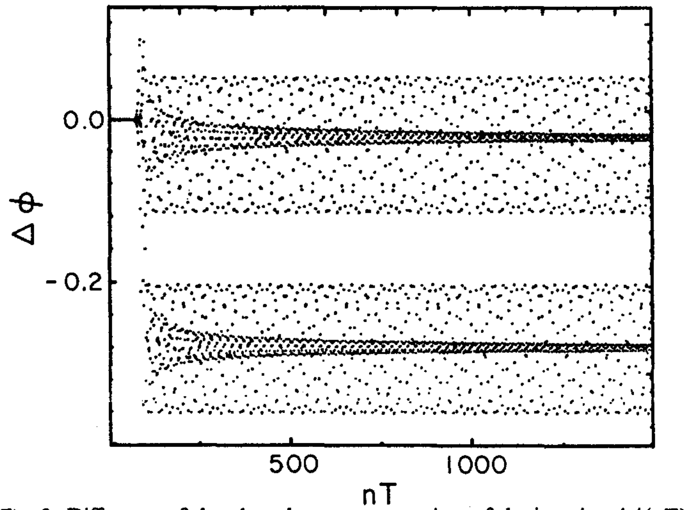

Fig. 2. Difference of the phase between two points of the junction $\Delta\phi(nT)$ vs. nT for a regime with a long transient in space and time at $\rho = 2.0079$ (inner bands) and a quasiperiodic regime at $\rho = 2.01$ (outer bands).

spatial symmetry is broken, much like the symmetry-breaking precursor (in phase space) of the usual transition to chaos. Figure 1(c) can be interpreted to be the inability of the system to sustain a soliton commensurate with its size, jumping instead into a sort of bifurcation in space. The system thus succeeds in breaking the symmetry in real space and not in phase space as in temporal chaos.

4. The two-frequency route to turbulence

In this section we present the development of turbulence as concerning the involvement of new degrees of freedom via the breakdown of coherence of the spatiotemporal profile.

As ρ is increased, once the breather forms from the initial flat condition there is long transient both in space and in time. The inner bands in Fig. 2 corresponds to a regime ($\rho = 2.0079$) that after such a long transient finally reaches the state with a breather oscillating with a frequency equal to half the driver (Fig. 1(c)).

As the system is driven harder the system ceases the long transient behavior in space and in time via generation of a quasiperiodic response. The outer bands in Fig. 2 presents for $\rho = 2.01$ the generation at the very onset of pattern formation of a breather-like excitation unlocked to the frequency of the driving force.

In Fig. 3(a) the continuous line represents the power spectrum of the voltage at the middle of the junction for the quasiperiodic regime at $\rho = 2.01$. The dashed lines in Fig. 3(a) represent the power spectrum of the phase difference $\Delta\phi$ between two points of the junction, this denotes in comparison with the local power spectrum that different points of the junction differ in the way the different frequencies linearly combine. This has the effect of reducing the coherence of the spatiotemporal profile.

Figure 3(b) presents local and two-point spectra for the critical attractor at the onset of disorder ($\rho \doteq 2.02$) while Fig. 3(c) corresponds to the already disordered regime ($\rho = 2.04$).

Figure 4(a–c) presents the phase space portraits for the same sequence of values of ρ that Fig. 3(a–c): they show the break-up of the torus into a disordered state. The attractor for the disordered state can now be regarded as a strange (chaotic) attractor: the activation of an increased number of

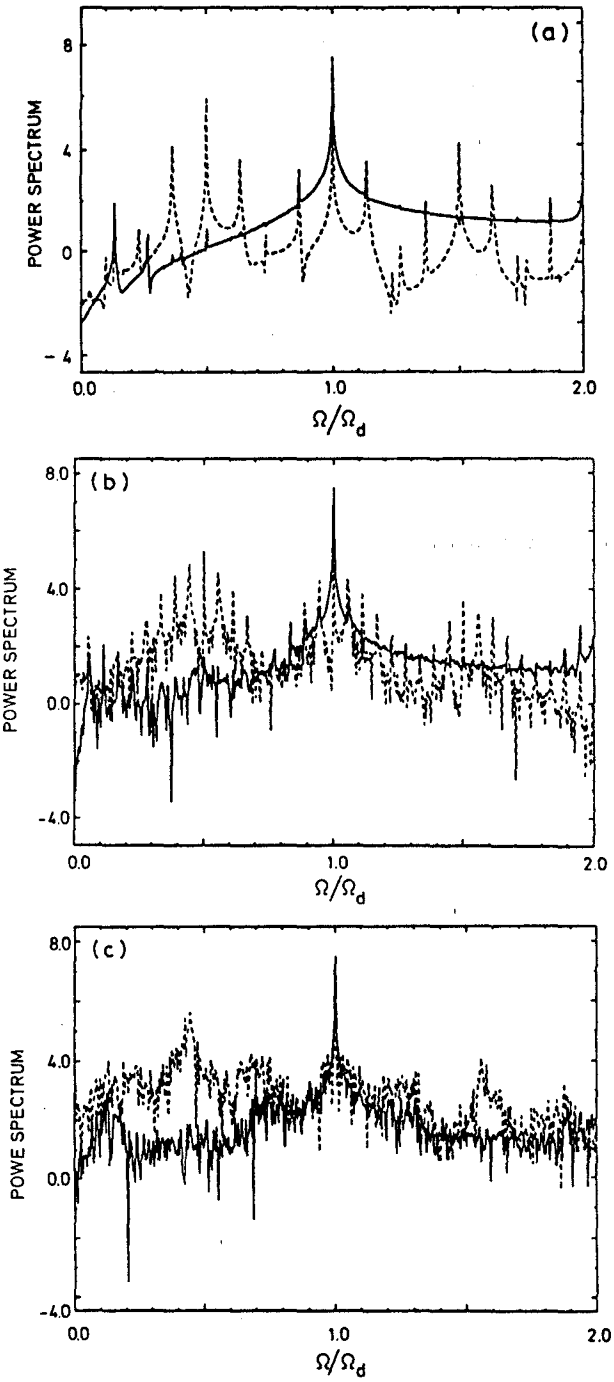

Fig. 3. Quasiperiodic transition to turbulence. The continuous line represents the power spectrum of the voltage ϕ_t at $x = L/2$. The dashed lines represent the power spectrum of the phase difference ($\Delta\phi(t) = \phi(L/2, t) - \phi(L', t)$). (a) $\rho = 2.01$, (b) $\rho = 2.02$, (c) $\rho = 2.04$.

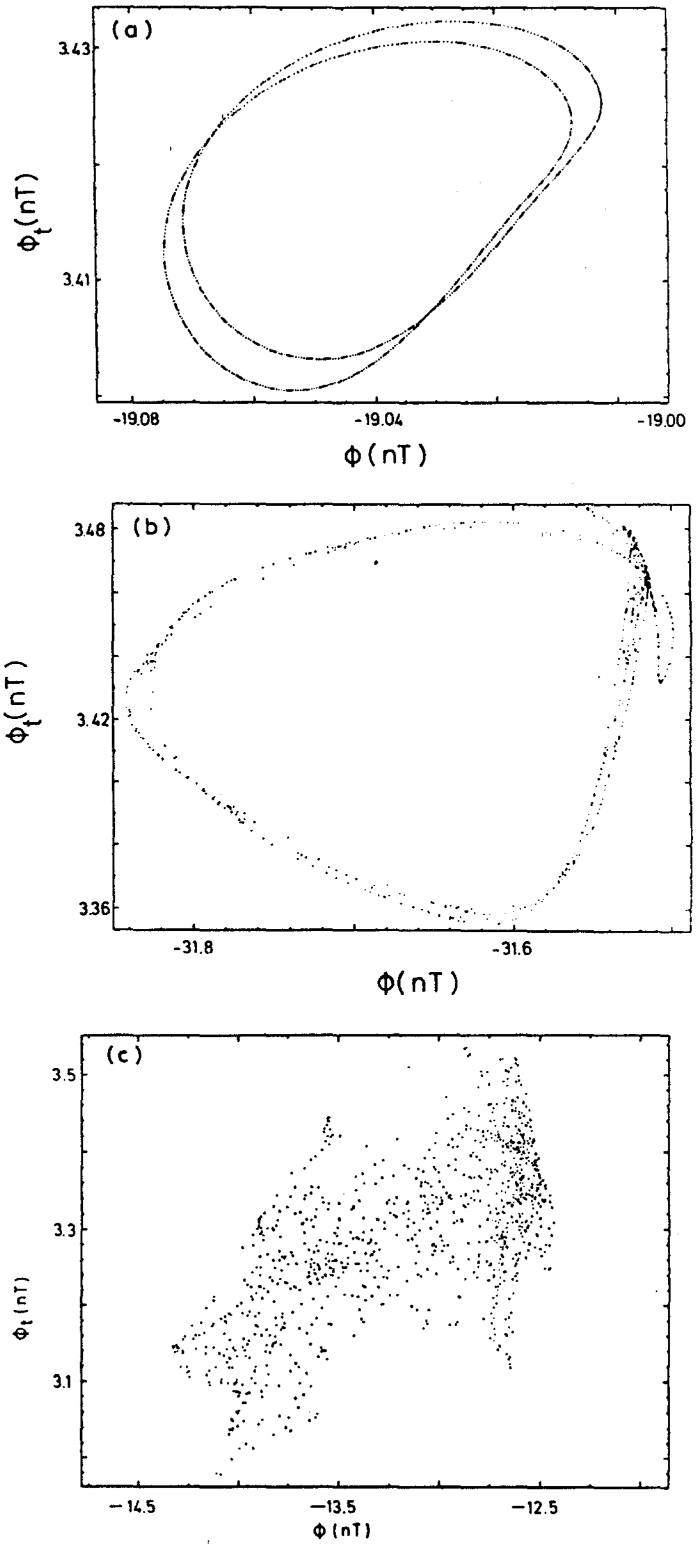

Fig. 4. Quasiperiodic transition to turbulence. Sequence of Poincaré maps $\phi_t(L/2, T)$ vs. $\phi(L/2, T)$. (a) $\rho = 2.01$, (b) $\rho = 2.02$, (c) $\rho = 2.04$.

degrees of freedom is responsible for the loss of fractality and autosimilarity (intimately related with the underlying mechanism of he chaotic phenomena).

The final state is disordered not only in time but in space. In Fig. 5 we present the coherence spectra [9] for the quasiperiodic regime at $\rho = 2.01$ (dashed lines) and for the disordered regime at ($\rho = 2.04$) (soid lines). We can appreciate that the coherence of the spatial profile decreases as the forcing is increased. Thus the resulting state at ($\rho = 2.04$) can be regarded as a *soft-turbulence regime*, a regime in which there is a breakdown of the pattern formation ability of the system [1, 7].

This soft-turbulent state contrasts with the *hard-turbulence regime* in which the pattern formation and conversion induces a reemergence of coherence in a narrow frequency range [1]. The LJJ is able to exhibit hard-turbulence when a magnetic field is applied [7].

5. Conclusions

We have shown that turbulent-like behavior can be autonomously excited in a one-dimensional solid state device, the LJJ, with a single oscillating drive by the competition between local (single degree of freedom) and global (collective) dynamics. We identified the spatiotemporal symmetry breaking, the

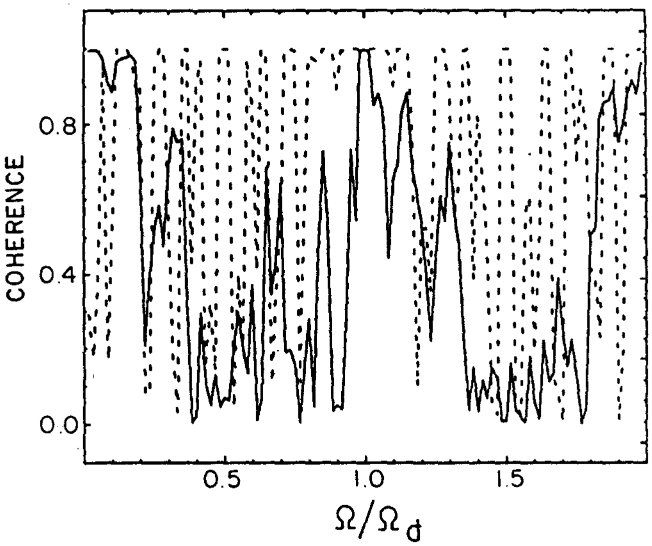

Fig. 5. Breakdown of coherence of the spatiotemporal profile. The dashed lines show the coherence spectrum for the quasiperiodic regime ($\rho = 2.01$) whereas the continuous line corresponds to a regime in which there is a breakdown of the pattern formation ability of the system ($\rho = 2.04$). Frequency is normalized to the drive frequency.

spontaneous pattern formation and the ability of the system to sustain a spatiotemporal profile unlocked to the driving force as intimately related to the transition to the turbulent state.

We have presented a novel route to turbulence which begins with a period doubling bifurcation accompanied with pattern formation. This rout differs from Feigenbaum's period doubling cascade as the control parameter is varied: the period 2 regime exhibits a direct transition to a two-frequency quasiperiodic regime and further increase of the control parameter destroys coherence in space. The knowledge of the differences between the onset of chaos and the onset of turbulence can provide a way to distinguish these two regimes in experimental situations.

Finally, we would like to remark that evidence for turbulent-like behavior in other solid-state systems has been reported [10].

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9. The coherence spectrum can reveal possible correlations in space for various frequencies. The coherence is defined as

$$C_{xy}(k) = \frac{|G_{xy}(k)|^2}{G_{xx}(k)G_{yy}(k)}$$

where G_{xx} and G_{yy} are the autopower spectra of the two real sequences and G_{xy} is the cross-power spectrum.
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