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Pattern control and suppression of spatiotemporal chaos using geometrical resonance

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Abstract

We generalize the concept of geometrical resonance to perturbed sine-Gordon, Nonlinear Schrödinger, φ^4, and Complex Ginzburg–Landau equations. Using this theory we can control different dynamical patterns. For instance, we can stabilize breathers and oscillatory patterns of large amplitudes successfully avoiding chaos. On the other hand, this method can be used to suppress spatiotemporal chaos and turbulence in systems where these phenomena are already present. This method can be generalized to even more general spatiotemporal systems. A short report of some of our results has been published in [Europhys. Lett. 64 (2003) 743].

1. Introduction

Spatiotemporal chaos [1–6] is one of the most important (and most studied) phenomena of recent years. Chaos can be advantageous in some situations, while in many others it should be avoided or controlled [7–15]. In certain cases, the desired effect is a high-amplitude periodic oscillation. We should drive a nonlinear system with a large external force to produce such a high-amplitude oscillation. However, this should be done in such a way that chaos is avoided. Different feedback mechanisms have been devised to control chaos [8,16–18]. A great deal of research has been dedicated also to the problem of suppresing chaos by harmonic (or just periodic) perturbations [12,15,19–26]. Among those works are the ones that use the concept of geometrical resonance (GR) [15,22,24–28].

In Ref. [15] the concept of GR was used as a chaos-eliminating mechanism for the perturbed φ^4 equation. In this paper we generalize the concept of geometrical resonance to a very general class of spatiotemporal systems which includes the sine-Gordon, Nonlinear Schrödinger, Boussinesq, Toda lattice and Complex Ginzburg–Landau equations (among others). We will use this concept as a method of chaos control when these equations are nonintegrable because of the presence of perturbations. GR is an extension of the linear notion of resonance to a nonlinear formulation based on a local energy conservation requirement [24].

The paper is organized as follows: In Section 2 we introduce the general concept of geometrical resonance for spatiotemporal systems. Section 3 is dedicated to the perturbed sine-Gordon equation. There, using the exact geometrical resonance condition for the perturbed sine-Gordon equation, we show how a breather can be stabilized using external perturbations. In Section 4 we show that an approximate geometrical resonance condition can control spatiotemporal chaos, introducing a given additional perturbation. In this case, the controlling perturbation is not

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arbitrary, only some parameters can be changed in order to control the dynamics. Section 5 is dedicated to the Nonlinear Schrödinger equation. In Section 6, using the φ^4 equation, we explain how this technique can be used even in systems where the original equation is not integrable. Section 7 is devoted to the Complex Ginzburg–Landau equation. Finally, in Section 8 we discuss some general conclusions and experimental considerations which include possible applications in new oil technologies.

2. Geometrical resonance

Let us consider the partial differential equation

$$K_0[\phi] + K_1[\phi, x] = q(x,t)P[\phi], \tag{1}$$

where $K_0[\phi]$ and $P[\phi]$ are functions of ϕ, and its derivatives: ϕ_t, ϕ_x, ϕ_{tt}, ϕ_{xx}, etc.

Equation

$$K_0[\phi] = 0 \tag{2}$$

is an integrable Hamiltonian system. This can be, for instance, the sine-Gordon equation (SGE) $\phi_{tt} - \phi_{xx} + \sin\phi = 0$, the Nonlinear Schrödinger equation (NLSE) $\mathrm{i}\phi_t + \phi_{xx} + 2|\phi|^2\phi = 0$, the Boussinesq equation $\phi_{tt} - \gamma\phi_{xx} - 2\alpha(\phi_x^2 + \phi\phi_{xx}) - \phi_{xxxx} = 0$, or the Toda lattice [29,30].

On the other hand, $K_1[\phi, x]$ includes dissipative terms and $q(x,t)P[\phi]$ is a very general driving force [6,31–33].

At GR, the amplitude, frequency, and space–time shape of $q(x,t)$ must satisfy some conditions so that some dynamical properties of the conservative system are preserved. We will call $\phi_{\mathrm{GR}}(x,t)$ a GR solution of Eq. (1) if

$$K_1[\phi_{\mathrm{GR}}, x] = q(x,t)P[\phi_{\mathrm{GR}}]. \tag{3}$$

This implies a local energy conservation requirement. The energy integral that is conserved for Eq. (2) is locally conserved for Eq. (1) if condition (3) holds. We can use this condition as a mechanism for chaos control when an additional condition holds: the GR solution must be an asymptotically stable solution of the (full) Eq. (1). This condition is introduced here for the first time. We will call Eq. (3) the exact GR condition and the solutions that satisfy this condition will be called GR solutions.

We can consider the energy of the system as a "local almost adiabatic invariant" [34]. Then we can write an approximate GR condition

$$\left\langle \frac{\mathrm{d}H}{\mathrm{d}t} \right\rangle_{T'} \simeq 0, \tag{4}$$

where H is the energy of the system and T' is the period of the chosen solution of Eq. (2).

3. Perturbed sine-Gordon equation

As an example, let us use the well-known driven and damped SGE

$$\phi_{tt} - \phi_{xx} + \gamma\phi_t + \sin\phi = q(x,t). \tag{5}$$

Suppose the task is to produce breathers of large amplitudes without entering a chaotic regime.

The exact breather solution to the unperturbed SGE is

$$\phi(x,t) = 4\arctan\left[\frac{\sqrt{1-\omega^2}\sin(\omega t)}{\omega\cosh[\sqrt{1-\omega^2}x]}\right], \tag{6}$$

where ω is arbitrary in the interval $\omega^2 < 1$.

The external force $q(x,t)$ satisfies the GR condition (3) when

$$q_{\mathrm{GR}}(x,t) = \frac{4\gamma\sqrt{1-\omega^2}\cos(\omega t)}{\cosh[\sqrt{1-\omega^2}x] + \left(\frac{1-\omega^2}{\omega^2}\right)\left(\frac{\sin^2(\omega t)}{\cosh[\sqrt{1-\omega^2}x]}\right)}. \tag{7}$$

In Eq. (5) if $q(x,t)$ is given by (7), the function (6) is an exact solution of the complete Eq. (5). When the parameters that define the perturbation (7) are fixed, there is only one frequency for which function (6) is the solution. This frequency is determined by that appearing in (7).

Let us investigate the stability of solution (6) in the framework of equation (5) with $q(x,t)$ given by function (7).

Suppose $\phi(x,t) = \phi_{br}(x,t) + \psi(x,t)$, where $|\psi(x,t)| \ll |\phi(x,t)|$. The equation for $\psi(x,t)$ will be

$$\psi_{tt} - \psi_{xx} + \gamma\psi_t + (\cos\phi_{br})\psi = 0. \tag{8}$$

This equation (for $\gamma = 0$) is well known, because it is obtained for the stability problem of the original breather solutions for the unperturbed sine-Gordon equation. From this equation we get that the breather is marginally stable. That is, the breather is stable in the sense of Lyapunov. For small $\psi(x,t=0)$, $\psi(x,t)$ will remain small. In fact, the solutions for $\psi(x,t)$ are time-periodic.

However, in the dissipative equation (8) (with $\gamma > 0$) the functions $\psi(x,t)$ will decay, i.e., $\psi(x,t) \to 0$ as $t \to 0$. In fact, if we define the "energy" of (8) as

$$E(\psi) = \frac{\psi_t^2}{2} + \frac{\psi_x^2}{2} + (\cos\phi_{br})\frac{\psi^2}{2}, \tag{9}$$

then,

$$\frac{\mathrm{d}E(\psi)}{\mathrm{d}t} = -\gamma\psi_t^2 < 0. \tag{10}$$

Nevertheless, the solution (6) is asymptotically stable in the framework of the full Eq. (5) with $q(x,t)$ given by function (7).

Thus, in the framework of the unperturbed SGE breathers form a continuum of solutions similar to the periodic solutions around the fixed points called centers in Dynamical Systems theory. These solutions are stable in the sense of Lyapunov but they are not asymptotically stable.

However, the breather solution (6) in the framework of Eq. (5) with $q(x,t)$ given by Eq. (7) is a spatiotemporal limit cycle. That is, this is an spatiotemporal attractor. All close initial conditions (in all space configurations) for $t \to \infty$ will tend to behave as this solution. This phenomenon is shown in Figs. 1 and 2.

Perturbation $q_{\mathrm{GR}}(x,t)$ can be approximated by a function of type $q(x,t) = f(t)g(x)$ where $g(x)$ is a bell-shaped function and $f(t)$ is a time-periodic function. This kind of perturbations has been used in several studies of the SGE [35,36].

The general study of Eq. (5) using the GR concept and the breather solutions leads to the following conclusions: We can avoid chaos with amplitudes A of $q(x,t)$ for which $|A| \leqslant 4\gamma\sqrt{1-\omega^2}$ and $\omega^2 < 1$. On the other hand, the range Q of the function $g(x)$ (i.e. the interval of x where $g(x)$ is not exponentially small) should be $Q \leqslant \frac{1}{\sqrt{1-\omega^2}}$.

In some cases, when these conditions are not satisfied, the breather is not stabilized and we can get an irregular behavior. Fig. 3 shows an example of the dynamics produced by Eq. (5) with $q(x,t) = A\cos(\omega t)\left[\cosh\frac{x}{Q} + \left(\frac{1-\omega^2}{\omega^2}\right)\left(\frac{\sin^2(\omega t)}{\cosh\frac{x}{Q}}\right)\right]^{-1}$ where $A = 4.5$, $\omega = 0.707$, $Q = 6.6$.

There is a wealth of works [35,36] dedicated to the numerical investigation of perturbed sine-Gordon equations using external forces of type $q(x,t) = f(t)g(x)$. All the results are in agreement with our theoretical results. For instance,

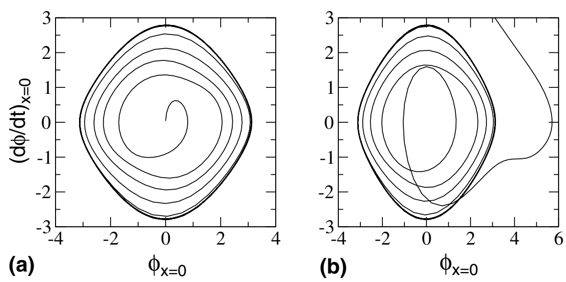

Fig. 1. The breather formed in Eq. (5) is a spatiotemporal attractor: (a) the initial condition are "inside" the spatiotemporal limit cycle ($\phi(x,0) = \phi_t(x,0) = 0$); (b) the initial conditions are outside the spatiotemporal limit cycle ($\phi(x,0) = 0, \phi_t(x,0) = 5.1/\cosh x$). In all cases $\gamma = 0.45$, $\omega = 0.707$.

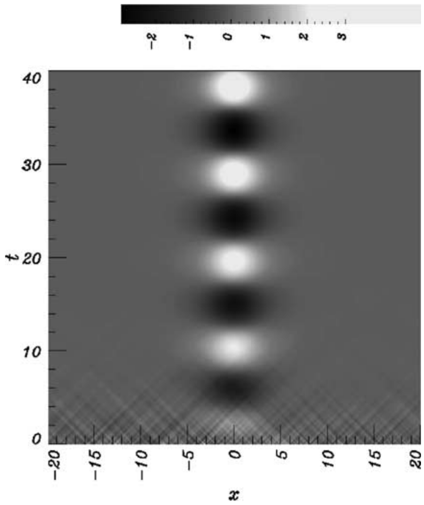

Fig. 2. Self-organization of the breather in Eq. (5). From random initial conditions the breather is reorganized ($\omega = 0.707$, $\gamma = 0.45$). In the numerical simulations with the discretized equation the initial conditions were produced by a pseudorandom number generator of uniformly distributed values in the interval $[-1, 1]$.

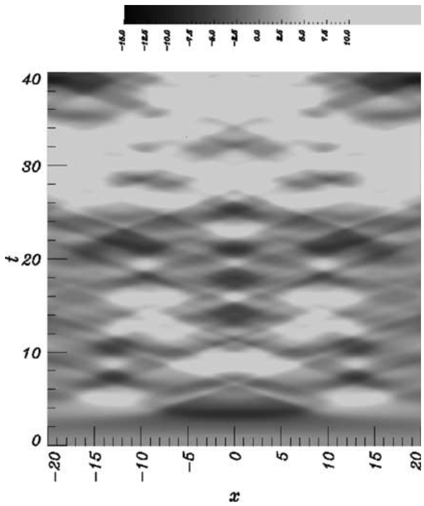

Fig. 3. Irregular dynamics produced with a perturbation where the amplitude A, the frequency ω and the range Q are not close to satisfy the GR condition ($A = 4.5$, $\omega = 0.707$, $Q = 6.6$).

following the ideas of Ref. [36] we studied the equation $\phi_{tt} + \gamma\phi_t - \phi_{xx} + \sin\phi = F(x,t)$ where $F(x,t) = 0$ if $0 \leqslant x \leqslant 15$, $F(x,t) = \eta_{ac}\cos\omega t$ ($\omega = 0.9$) if $15 < x < 25$, and $F(x,t) = 0$ if $25 \leqslant x \leqslant 40$. For $\eta_{ac} = 0.01$ and $\gamma = 0.01$, we obtain a nonchaotic regime. For $\eta_{ac} = 0.5$ and $\gamma = 0.01$, we obtain spatiotemporal chaos.

4. Using the approximate condition

For the perturbed SGE

$$\phi_{tt} - \phi_{xx} + R(\phi, \phi_t, x) + \sin\phi = q(x,t), \tag{11}$$

Eq. (4) becomes

$$\left\langle \int_{-\infty}^{\infty} [R(\phi, \phi_t, x)\phi_t - q(x,t)\phi_t]\,\mathrm{d}x \right\rangle_{T'} = 0. \tag{12}$$

Sometimes we have the task of suppressing chaos using a given harmonic perturbation. Consider e.g. the following equation:

$$\phi_{tt} - \phi_{xx} + \gamma\phi_t + \sin\phi = f_0 \sin(\omega_d t) + f_c \sin[\omega_c t + \theta], \tag{13}$$

where $f_0 \sin(\omega_d t)$ is a chaos-producing excitation, while $f_c \sin[\omega_c t + \theta]$ is a chaos-suppressing excitation. The parameters of the chaos-suppressing excitation should be determined.

In this case we can use the condition (12) to find parameters for the chaos-suppressing perturbation. Fig. 4 shows an example of chaos control using this technique.

We would like to stress here that the force (7) can be used to control a well-developed spatiotemporal chaos.

Let us consider the following equation:

$$\phi_{tt} - \phi_{xx} + \gamma\phi_t + \sin\phi = f_0 \sin(\omega_d t) + F_c(x,t). \tag{14}$$

When $F_c(x,t) \equiv 0$, the system presents spatiotemporal chaos for $-\infty < x < \infty$. Now, if we turn on the controlling force $F_c(x,t)$ defined as function (7), we obtain a very regular spatiotemporal pattern as that shown in Fig. 5.

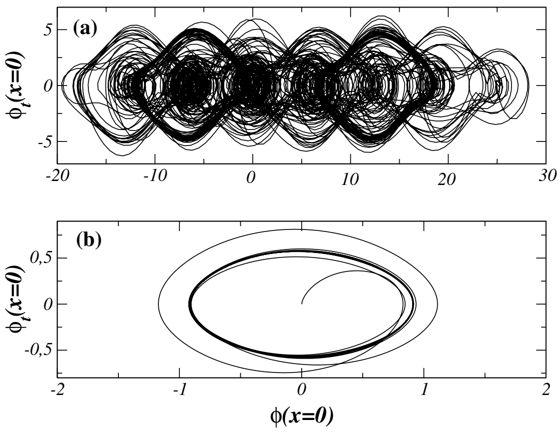

Fig. 4. Suppression of spatiotemporal chaos using a given chaos-suppressing excitation in Eq. (13) ($f_0 = 0.91, \omega_d = 0.6$): (a) well-developed spatiotemporal chaos when $f_c = 0$ and (b) controlled dynamics with $f_c = 0.4$, $\omega_c = 0.6$, $\theta = \pi/2$.

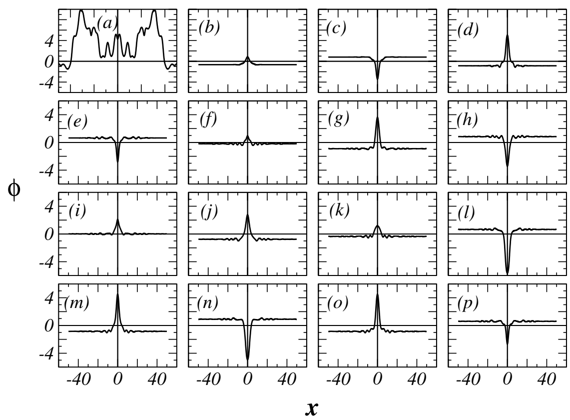

Fig. 5. Controlling spatiotemporal chaos using a localized excitation in Eq. (14): (a) spatial profile for a given time moment corresponding to spatiotemporal chaos when $F_c(x,t) = 0$ and (b)–(p) spatial profiles for different time instants when $F_c(x,t)$ is given by Eq. (7), $\gamma = 0.1$, $f_0 = 0.5$, $\omega_d = 0.6$, $\omega = 0.6$.

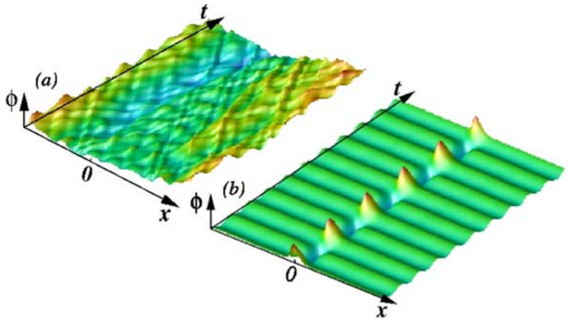

Fig. 6. Chaos and order in the spatiotemporal dynamics of Eq. (15): (a) well-developed chaos, $F_p(x,t) = [f_1\cos(\omega_1 t)g_1(x) + f_2\cos(\omega_2 t)g_2(x)]Q(x)$, $g_1(x) = \cosh[B_2(x-x_2)]$, $g_2(x) = \cosh(B_1 x)$, $Q(x) = \mathrm{sech}(B_1 x)\mathrm{sech}[B_2(x-x_2)]$, $f_1 = 0.5$, $\gamma = 0.01$, $\omega_1 = 1$, $B_1 = 0.01$, $x_2 = 0.5$, $f_2 = 4\gamma\sqrt{1-\omega_c^2}$, $B_2 = \sqrt{1-\omega_2^2}$, $\omega_2 = 0.5$ and (b) regular dynamics obtained when the controlling function is applied, $F_c(x,t) = f_c\cos(\omega_c + \theta)/\cosh(B_1 x)$, $f_c = 0.47$, $\omega_c = 1$.

The most important remark here is that we are controlling spatiotemporal chaos in the whole space using a localized perturbation. Also, we should add here that other works have used localized perturbations to control spatiotemporal chaos [37–39].

In general, even if we have a very complicated chaos-producing perturbation $F_p(x,t)$ as in the following equation:

$$\phi_{tt} - \phi_{xx} + \gamma\phi_t + \sin\phi = F_p(x,t) + F_c(x,t), \tag{15}$$

the important idea is to use a controlling function $F_c(x,t)$ such that the general force $F(x,t) = F_c(x,t) + F_p(x,t)$ is as close as possible to a $q_{\mathrm{GR}}(x,t)$ that satisfies the GR condition. This can be for instance the function defined by Eq. (7).

An example is shown in Fig. 6.

5. Nonlinear Schrödinger equation

The damped and ac-driven NLSE

$$\mathrm{i}\phi_t + \phi_{xx} + 2|\phi|^2\phi + \mathrm{i}\alpha u = \varepsilon \mathrm{e}^{\mathrm{i}\omega t} \tag{16}$$

is another fundamental model in many areas of Physics [40–42]. At sufficiently large ε the dynamics of this model becomes chaotic [42].

Suppose we have a general driving term

$$\mathrm{i}\phi_t + \phi_{xx} + 2|\phi|^2\phi + \mathrm{i}\alpha u = q(x,t). \tag{17}$$

We will take the one-soliton solution of unperturbed NLSE [30] as a GR solution: Then $q(x,t)$ must satisfy the condition:

$$q_{\mathrm{GR}}(x,t) = \frac{\alpha\sqrt{\omega}e^{\mathrm{i}(\omega t + \pi/2)}}{\cosh(\sqrt{\omega}x)}. \tag{18}$$

where ω can be any positive number.

Thus, if the perturbation is localized and the amplitude ε satisfies the condition $\varepsilon \approx \alpha\sqrt{\omega}$, then the chaotic regime can be avoided. The one-soliton solution of NLSE is stabilized. We should remark that this can be achieved also by other localized perturbations.

If we use the two-soliton breather solution as a GR solution, we can obtain another driving force satisfying a GR condition:

$$q_{\mathrm{GR}}(x,t) = \frac{4\alpha[\cosh(3x) + 3e^{\mathrm{i}8t}\cosh(x)]\mathrm{e}^{\mathrm{i}(t+\pi/2)}}{\cosh(4x) + 4\cosh(2x) + 3\cos(8t)}. \tag{19}$$

See Ref. [30] for a discussion of multisoliton solutions. As in Eq. (18) we can introduce here an arbitrary parameter ω. However, in this case, we are more interested in the relationship between the two intrinsic frequencies of the solution.

This force is similar to a function of type

$$F(x,t) = \varepsilon_1 g_1(x) e^{i(\omega_1 t + \pi/2)} + \varepsilon_2 g_2(x) e^{i(\omega_2 t + \pi/2)}, \tag{20}$$

where $\omega_1 = 1$, $\omega_2 = 9$, and $g_1(x)$ and $g_2(x)$ are localized functions.

In Ref. [42] a breather was stabilized using a two-frequency drive:

$$F(x,t) = \varepsilon_1 e^{i\omega_1 t} + \varepsilon_2 e^{i\omega_2 t}, \tag{21}$$

where $\omega_1 = 1$ and $\omega_2 = 9$. This result can be seen as a confirmation of the GR approach for the NLSE. We should add that the phenomenon of breather stabilization is quite robust. For instance, if $\omega_1 = 1$, in addition to $\omega_2 = 9$, other close frequencies can be used, namely $\omega_2 = 8$ and $\omega_2 = 10$. This means that the GR condition can be satisfied approximately and that Eq. (4) can also be used as a guide for the search of a controlling force. As in the case of the breather of the SGE, the breather (19) of the NLSE is asymptotically stable.

Regarding localized excitations we should emphasize that GR analysis explains diverse fundamental results on stability of localized solutions previously obtained by perturbation theory [6,15,42] including those relative to one-soliton and two-soliton solutions of the NLSE. In this sense, in future works, it would be interesting to consider the case of the N-soliton solutions (see e.g. Appendix B, Ref. [30]).

6. Perturbed ϕ^4 equation

The general theory for Eq. (1) assumes that Eq. (2) is an integrable system. In this section we wish to show an example where, even if the basic equation is nonintegrable, we can apply the concept of geometrical resonance.

The well-known ϕ^4 equation

$$\phi_{tt} - \phi_{xx} - \frac{1}{2}(\phi - \phi^3) = 0 \tag{22}$$

is nonintegrable in the sense that only a finite number integrals of motion are conserved. However, this does not mean that all the regimes in Eq. (22) are chaotic. In fact, the behavior of a kink-soliton in Eq. (22) is similar to that of a free particle [31–33].

Moreover, even a system much more complicated than Eq. (22)

$$\phi_{tt} - \phi_{xx} - \frac{1}{2}(\phi - \phi^3) = F(x) \tag{23}$$

can be shown to be equivalent to a very regular particle motion.

If we have a kink-soliton as the initial condition for Eq. (23), then the dynamics will be similar to a particle moving in a potential where $F(x)$ is the effective force [15,31–33]. The zeroes of $F(x)$ are approximately the equilibrium positions for the soliton. In fact, if $x = x_0^*$ is an equilibrium position ($F(x_0^*) = 0$), then it is stable if $\left(\frac{\partial F(x)}{\partial x}\right)_{x=x_0^*} > 0$. Small perturbation of a soliton near a stable equilibrium position lead to linear oscillations of the soliton center of mass. These oscillations are not chaotic.

It is well known [31–33] that the following perturbed ϕ^4 equation

$$\phi_{tt} + \gamma\phi_t - \phi_{xx} - \frac{1}{2}(\phi - \phi^3) = A\tanh(Bx) \tag{24}$$

with a kink-soliton as the initial condition, is equivalent to a damped harmonic oscillator in the sense that the center of mass of the soliton performs linear damped oscillations around the point $x = 0$. Thus the system

$$\phi_{tt} - \phi_{xx} - \frac{1}{2}(\phi - \phi^3) = A\tanh(Bx) \tag{25}$$

can be seen as a soliton "harmonic" oscillator where the soliton motion is "integrable".

Chaos can be produced using the following perturbations in the ϕ^4 equation

$$\phi_{tt} + \gamma\phi_t - \phi_{xx} - \frac{1}{2}(\phi - \phi^3) = F(x) - P_0\frac{\cos(\omega t)}{\cosh^2(Bx)}, \tag{26}$$

where $F(x)$ is a function with three zeroes and $P_0 \cos(\omega t)/\cosh^2(Bx)$ is an additional spatiotemporal perturbation [6,15,31–33].

Some other forces $F(x)$ also act as effective nonlinear forces for the soliton. For instance, $F(x) = A\tan^3(Bx)$ leads to a soliton motion equivalent to the cubic Duffing equation

$$X_{tt} + \gamma X_t + bX^3 = f_0 \cos(\omega t). \tag{27}$$

Consider now the following chaos-suppression problem:

$$\phi_{tt} + \gamma\phi_t - \phi_{xx} - \frac{1}{2}(\phi - \phi^3) = F(x) - P_0 \frac{\cos(\omega t)}{\cosh^2(Bx)} + F_c(x,t). \tag{28}$$

For $F_c(x,t) \equiv 0$, the soliton in Eq. (28) enters a regime of chaotic motion. We assume that the controlling function is given by

$$F_c(x,t) = \frac{g_c \cos(\omega_c t + \theta_c)}{\cosh^{\Lambda}(Bx)}. \tag{29}$$

The parameters of $F_c(x,t)$ can be obtained from the application of the approximate GR condition:

$$\left\langle \int_{-\infty}^{\infty} \left[-\gamma\left(\frac{\partial\phi_{\mathrm{GR}}}{\partial t}\right)^2 - P_0 \frac{\partial\phi_{\mathrm{GR}}}{\partial t} \frac{\cos(\omega t)}{\cosh^2(Bx)} + g_c \frac{\partial\phi_{\mathrm{GR}}}{\partial t} \frac{\cos(\omega_c t + \theta_c)}{\cosh^{\Lambda}(Bx)} \right] \mathrm{d}x \right\rangle_{T'} = 0. \tag{30}$$

The approximate solution for the "unperturbed" nonchaotic soliton motion in equation

$$\phi_{tt} - \phi_{xx} - \frac{1}{2}(\phi - \phi^3) = F(x) \tag{31}$$

is the following:

$$\phi_{\mathrm{GR}}(x,t) = A\tanh(Bx) + \frac{h_{00}\cos(\omega_0 t + \theta_0)}{\cosh^{\Lambda}(Bx)}, \tag{32}$$

where $\Lambda(\Lambda+1) = 3A^2/2B^2$, $\omega_0^2 = \Gamma_0 \equiv B^2\Lambda - 1$.

We will use this function as our geometrical resonance solution. Eq. (30) yields

$$-\pi\gamma h_{00} - g_c \int_0^{T_0} \sin(\omega_0 t + \theta_0)\cos(\omega_c t + \theta_c)\,\mathrm{d}t + \pi C_{\Lambda} P_0 \int_0^{T_0} \sin(\omega_0 t + \theta_0)\cos(\omega t)\,\mathrm{d}t = 0, \tag{33}$$

where $T_0 = 2\pi/\omega_0$ and $C_{\Lambda} = [\int_0^{\infty} \mathrm{sech}^{\Lambda+2}(x)\,\mathrm{d}x][\pi \int_0^{\infty} \mathrm{sech}^{2\Lambda}(x)\,\mathrm{d}x]^{-1}$. We can play with the control parameters g_c, ω_c and θ_c in such a way that Eq. (33) is satisfied (at least approximately). Eq. (33) can be re-written in the following form:

$$\frac{g_c}{\pi} = \frac{C_{\Lambda} P_0 \Psi(\beta, \theta_0, 0) - \gamma h_{00}\omega_0}{\Psi(\alpha, \theta_0, \theta_c)}, \tag{34}$$

where $\alpha \equiv \omega_c/\omega_0$, $\beta \equiv \omega/\omega_0$, $\Psi(\lambda, \theta, \delta) \equiv \int_0^{2\pi} \sin(t+\theta)\cos(\lambda t + \delta)\,\mathrm{d}t$. The set of parameters $\{\omega, P_0, \beta, A, h_{00}, \theta_0, \gamma\}$ is given.

Eq. (34) produces the conditions to be satisfied by the control parameters. Given $\alpha = p/q$, with p and q relative primes, we can use a θ_c^* that maximizes $\Psi(\alpha, \theta_0, \theta_c)$.

Once θ_c^* is obtained, we will have $g_{c(\min)}$:

$$g_{c(\min)} = \frac{\pi[C_{\Lambda} P_0 \Psi(\beta, \theta_0, 0) - \gamma h_{00}\omega_0]}{\Psi(\alpha, \theta_0, \theta_c^*)}. \tag{35}$$

7. Complex Ginzburg–Landau equation

The control of spatiotemporal chaos (or turbulence) in the Complex Ginzburg–Landau equation (CGLE) [13,43–48] is a problem of great practical interest [48].

We are interested in the modified CGLE [13,44]:

$$\phi_t = \phi + (1 + \mathrm{i}c_1)\phi_{xx} - (1 - \mathrm{i}c_3)|\phi|^2\phi + F_c(x,t). \tag{36}$$

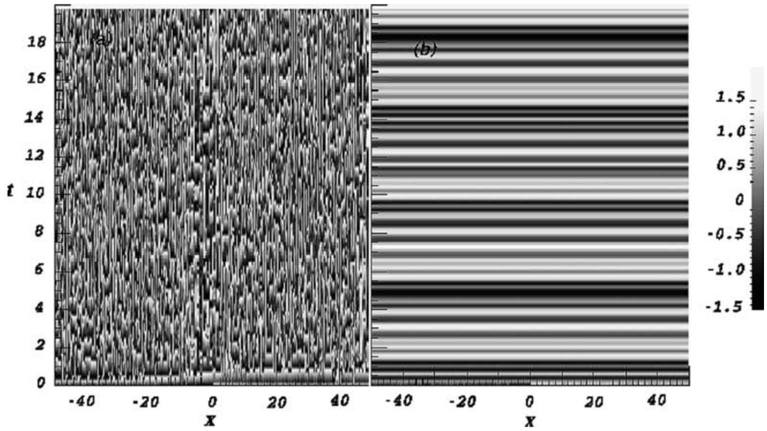

Fig. 7. Phase dynamics in Eq. (36). (a) Phase turbulence for $c_1 = 2$, $c_3 = 0.8$, $F_c(x,t) = 0$ and (b) suppression of turbulence when control perturbation (38) is applied, $\omega = 12$.

The term $F_c(x,t)$ is the control signal. Without the control signal ($F_c(x,t) = 0$), the turbulence develops when the Benjamin–Feir condition $1 - c_1c_3 < 0$ is satisfied.

This equation can be rewritten in the following form:

$$i\phi_t + c_1\phi_{xx} + c_3|\phi|^2\phi = i(\phi_{xx} + \phi - |\phi|^2\phi) + iF_c(x,t). \tag{37}$$

When the right-hand side of Eq. (37) is zero, it reduces to the NLSE.

If $\phi(x,t) = f(x)\exp(-i\omega t)$ is a soliton solution of the NLSE, then we can use the following controlling signal:

$$F_c(x,t) = [f^3(x) - f(x) - f_{xx}(x)]\exp(-i\omega t). \tag{38}$$

Eq. (36) (with $F_c \equiv 0$) presents turbulence for $c_1 = 2$, $c_3 = 0.8$. We have been able to suppress this turbulence using the $F_c(x,t)$ given by Eq. (38) with $\omega = 12$ and $f(x)$ is the one-soliton solution of equation $c_1f_{xx} - \omega f + c_3f^3 = 0$ [30]. Fig. 7 shows the behavior of (36) before and after the application of the controlling signal (38).

In this context, we should explain that in some cases, the stabilization process can require a force that is not a small perturbation. Furthermore, this technique can be used both as a way to stabilize a pre-existing solution of the unperturbed system and as a way to impose an arbitrary solution to the system. However, the success of all these endeavors depends on a very important fact: the final solution should be an asymptotical stable solution of the perturbed system. Incidentally, we should mention that the stabilization of unstable plane waves in the CGLE can be done using a nonlinear diffusion term [49].

8. Discussion and conclusions

In some situations we can apply some perturbations using technological means in order to satisfy the stability conditions. Nevertheless, we should say that, very often, nature itself can apply the controlling perturbations. There are many natural regimes described by the mentioned equations in the presence of perturbations where the resulting dynamics is not chaotic. Our results can provide an explanation for these phenomena.

Numerous observations and experiments show that elastic waves from natural phenomena and human-made machines may alter water and oil production [50]. In some cases wave excitation may appreciably increase the mobility of these fluids. A new technology [50] based on these experiments is used to stimulate the reservoir as a whole. Here seismic frequency waves are applied at the earth's surface by arrays of vibrators. Many of the phenomena involved in this effect are described by the equations discussed in this paper, namely: NLSE, SGE, Boussinesq equation and other equations of type (1) (see Ref. [51] and references therein). For the optimization of the method, it is necessary to sustain spatiotemporal nonlinear oscillations of the reservoir with some frequency and shape. Based on ideas related to the results presented in this paper we have designed a new technology using a specific geometrical arrangement of the surface vibrators. Further details will be presented elsewhere [52]. However this is just one example of the feasibility of implementing experimentally some of the paper's results.

Another example for the application of this theory is the following. The nonlinear PDE possess an infinite number of different solutions. Among them one can choose a feasible one in order to implement our technique.

Even if only a given type of perturbation is allowed due to technical limitations, it is always possible to use the approximate condition (4) as in the case of task (13).

The concept that links all the situations where we have been able to suppress chaos is based on the mutual cancellation of nonintegrable terms as described by Eqs. (3) and (4). In other words, we should add some temporal perturbation in such a way that (at least approximately) both the dissipative and the total driving terms mutually cancel. A remarkable situation (which is a particular case of the general theory but, at the same time, is present in all the studied systems) is that of breather-like oscillations. These patterns can be stabilized using some spatially localized time-periodic perturbations, where the amplitude, the spatial range and the frequency must satisfy some relationship. However, this phenomenon is robust. A fine-tuning is not necessary. There is always a whole valid interval of values for the amplitude, range and frequency that produces qualitatively the same result.

In conclusion, we have shown that the waveform of the perturbations in spatiotemporal nonlinear systems is crucial for the resulting dynamics. This can be seen in the fact that two periodic time-dependent perturbations with the same amplitude and frequency can produce different effects if the waveforms are different. However, in spatiotemporal systems we should also consider the space-dependent part of the waveform. The most common perturbation in scientific research is $F(t) = f_0 \cos(\omega t)$. However, nature is very rich in dynamical behaviors. Our work shows that using very general spatiotemporal perturbations $F(x,t)$ we can make the difference between regular or chaotic behavior.

Using certain spatiotemporal perturbations $F(x,t)$ we can stabilize a breather or we can produce a turbulent dynamics. We have been able to control different patterns in the sine-Gordon, Nonlinear Schrödinger, and Complex Ginzburg–Landau equations. Each of these systems possesses wide applications in many areas of Physics. Furthermore we believe that these ideas can be applied to other spatiotemporal systems.

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